Math 2 HW # 2

1. Let X be a random variable with density

$$f_X(t) = \begin{cases} Cx^{-3} & \text{if } x > 5 \\ 0 & \text{else.} \end{cases}$$

   (a) Calculate the value of the constant C so that f_X is a probability density.

   (b) Calculate the cumulative distribution function for X.

   (c) Calculate the expectation value of X.

   (d) Show that the expectation value of $E[X^2] = \infty$. Conclude from this that also $Var(X) = \infty, \sigma(X) = \infty$.

2. Consider the following gambling game: player A pays n dollars to player B. Then player A flips a fair coin, if it's "heads" he flips again; he keeps flipping until it falls "tails". Then player B pays 2^k dollars to player A, where k is the numbers of flips that were "heads".

   (a) Show that the expected value for player B's profit for one game is $-\infty$.

   (b) Suppose that the following additional rule is added to the game: if A suceeds in flipping 40 heads in a row, then the game ends. Calculate the expected value of the new game for player B.

   (c) Despite the expected profit of $-\infty$, most mathematicians would play the original game as B if n is sufficiently large, say $n = 40$. Do you think this is a good idea, and why/ why not?

3. (Problem 5.21E from *A first course in probability* by Sheldon Ross; exact wording altered) Suppose that the height of a man is a normal random variable with parameters (measured in inches) $\mu = 71, \sigma = 2.5$. According to this model, calculate the percentage of men taller than $74''$, and $77''$.

4. (Problem 5.15T from *A first course in probability* by Sheldon Ross; exact wording altered) Suppose X is uniformly distributed over $(0, a)$. Calculate the hazard rate function for X.

5. (Problem 5.6ST from *A first course in probability* by Sheldon Ross; exact wording altered) Your company must make a sealed bid agains one other company for a construction project. If your bid succeeds, it will cost you $100,000$ dollars to complete the work. You model the other company's bid as a random variable with uniform distribution on the interval from $70,000$ to $140,000$ dollars. How much should you bid to maximize expected profit?